

6th Week Wednesday- Vector Spaces

29 Mart 2023 Carsamba 15:09

Linear Algebra

matrices JLE vectors, pair

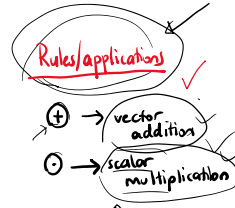
Recall from physics:

$\mathbb{R} \in \mathbb{R}$ → more than this

standard vector addition → $(x_1, y_1) + (x_2, y_2) = (x_1+x_2, y_1+y_2)$

standard scalar multiplication → $r \cdot (x_1, y_1) = (rx_1, ry_1)$

more than this



Decision (Democratic)

10 properties

Vector Space?

Is (V, \oplus, \odot) a vector space or not?

- n-tuples $n=1,2,\dots,n$
- matrices $m \times n$
- polynomials degree less than n

Ex $\mathbb{R}^3, \oplus, \odot$

$(x_1, y_1, z_1) \oplus (x_2, y_2, z_2) = (x_1+x_2, y_1+y_2, z_1+z_2)$
 $r \odot (x, y, z) = (rx, ry, rz) \in \mathbb{R}^3$

$\forall \vec{v} \in V, \forall r \in \mathbb{R}$

- 1) $\vec{v}_1 \oplus \vec{v}_2 \in V \rightarrow$ closed property of \oplus on V
- 2) $r \odot \vec{v} \in V \rightarrow$ " " " \odot on V

Recall

\oplus	Δ	1	2	3	4	\odot
$1 \Delta 2 = 3$	$\rightarrow 1$	4	3	1	2	
$2 \Delta 1 = 2$	$\rightarrow 2$	2	1	4	3	
	3	1	2	3	4	
	4	4	2	1	3	

$3 \Delta 1 = 5 \notin V$ not closed

Ex $\mathbb{R}^2, \oplus, \odot$

$(x_1, y_1) \oplus (x_2, y_2) = (x_1/x_2, y_1/y_2)$ ← is \oplus closed on \mathbb{R}^2 ? X

$r \odot (x, y) = (rx, ry)$

Counter example: $(3, 5) \oplus (0, -8) = (3/0, -40) \notin \mathbb{R}^2$

$\Rightarrow (\mathbb{R}^2, \oplus, \odot)$ is not a vector space.

- 1) $\vec{v}_1 \oplus \vec{v}_2 = \vec{v}_2 \oplus \vec{v}_1 \rightarrow$ commutativity of \oplus
- 2) $\vec{v}_1 \oplus (\vec{v}_2 \oplus \vec{v}_3) = (\vec{v}_1 \oplus \vec{v}_2) \oplus \vec{v}_3 \rightarrow$ associativity of \oplus
- 3) $\vec{0}_V \in V, \vec{v} \oplus \vec{0}_V = \vec{v}$ identity element of $\oplus \in V$
- 4) $\vec{v} \in V, \vec{v} \oplus \vec{v}^{-1} = \vec{0}_V$ inverse element of \oplus for any vector $\in V$

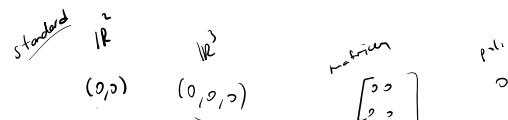
Ex $\mathbb{R}^3, \oplus, \odot$

$(x_1, y_1, z_1) \oplus (x_2, y_2, z_2) = (x_1-x_2, y_1+2y_2, z_1z_2)$
 $r \odot (x, y, z) = (rx, ry, rz)$

$(x_1, y_1, z_1) \oplus (x_2, y_2, z_2) = (x_2, y_2, z_2) \oplus (x_1, y_1, z_1)$
 $(x_1-x_2, y_1+2y_2, z_1z_2) = (x_2-x_1, y_2+2y_1, z_2z_1)$

Counter example: $(1, 2, 3) \oplus (-1, 3, 4) = (1-(-1), 2+2 \cdot 3, 3 \cdot 4) = (2, 8, 12)$
 $(-1, 3, 4) \oplus (1, 2, 3) = (-1-(1), 3+2 \cdot 2, 4 \cdot 3) = (-2, 7, 12)$

\oplus is not commutative.
 $\Rightarrow (\mathbb{R}^3, \oplus, \odot)$ is NOT vector space.



$\vec{0}, \odot, \forall r, s \in \mathbb{R}$

1) $r \odot (\vec{v}_1 \oplus \vec{v}_2) = (r \odot \vec{v}_1) \oplus (r \odot \vec{v}_2) \rightarrow$ Distributivity [D-1]

2) $(r+s) \odot \vec{v} = (r \odot \vec{v}) \oplus (s \odot \vec{v}) \rightarrow$ " [D-2]

3) $(rs) \odot \vec{v} = r \odot (s \odot \vec{v}) \rightarrow$ [D-3]

4) $1 \in \mathbb{R}, 1 \odot \vec{v} = \vec{v} \rightarrow$ identity element of \odot

$\mathbb{R}^2, +, \cdot$ is a vector space.
 Standard $\oplus \rightarrow (x_1, y_1) + (x_2, y_2) = (x_1+x_2, y_1+y_2)$
 Standard $\odot \rightarrow r(x, y) = (rx, ry)$
 within triples \mathbb{R}^3 is a vector space.
 $\mathbb{R}^n, +, \cdot$ is a vector space.
 within n-tuples

$\mathbb{R}^{m \times n}, \oplus, \odot$
 \oplus : addition of $m \times n$ matrices
 \odot : scalar multiplication on matrices
 is a vector space.

\mathbb{P}_n : the set of polynomials of degree less than n
 $= \{ a_0 + a_1x + a_2x^2 + \dots + a_{n-1}x^{n-1} : a_i \in \mathbb{R} \}$
 \oplus : polynomial addition
 \odot : scalar multiplication on polynomials.
 is a vector space.

10. Let S be the set of all ordered pairs of real numbers. Define scalar multiplication and addition on S by

\mathbb{R}^2
 $\alpha(x_1, x_2) = (\alpha x_1, \alpha x_2)$
 $(x_1, x_2) \oplus (y_1, y_2) = (x_1 + y_1, 0)$

$(x_1, x_2) \oplus (0, 0) = (x_1, x_2)$

- closed \odot 1) $\vec{v}_1 \oplus \vec{v}_2 \in V \rightarrow$ closed property of \oplus on V ✓
- 2) $r \odot \vec{v} \in V \rightarrow$ " " " \odot on V ✓

- identity \oplus 1) $\vec{v}_1 \oplus \vec{v}_2 = \vec{v}_2 \oplus \vec{v}_1 \rightarrow$ commutativity of \oplus ✓
- 2) $\vec{v}_1 \oplus (\vec{v}_2 \oplus \vec{v}_3) = (\vec{v}_1 \oplus \vec{v}_2) \oplus \vec{v}_3 \rightarrow$ associativity of \oplus ✓
- identity \odot 3) $\vec{0}_V \in V \vec{v} \oplus \vec{0}_V = \vec{v}$ identity element of $\oplus \in V$ X
- inverse \odot 4) $-\vec{v} \in V \vec{v} \oplus -\vec{v} = \vec{0}_V$ inverse element of \oplus for any vector $\in V$ X

- \rightarrow 1) $r \odot (\vec{v}_1 \oplus \vec{v}_2) = (r \odot \vec{v}_1) \oplus (r \odot \vec{v}_2) \rightarrow$ Distributivity [D-1] ✓
- \rightarrow 2) $(r+s) \odot \vec{v} = (r \odot \vec{v}) \oplus (s \odot \vec{v}) \rightarrow$ " [D-2] X
- 3) $(rs) \odot \vec{v} = r \odot (s \odot \vec{v}) \rightarrow$ [D-3]
- 4) $1 \in \mathbb{R} 1 \odot \vec{v} = \vec{v} \rightarrow$ identity element of \odot

$r=2 \quad s=3 \quad v = (-1, 4)$
 $(r+s) \odot \vec{v} = 5 \odot (-1, 4) = (-5, 20)$
 $(r \odot \vec{v}) \oplus (s \odot \vec{v})$
 $(-2, 8) \oplus (-3, 12) = (-5, 0)$